FST 3-8 Notes
Topic: Inverses of Functions
GOAL:
Define inverse of a function, discuss how to determine an inverse of a function from its graph or by graphing, and then examine the skill of finding an equation of an inverse.

SPUR Objectives

B Find inverses of functions.

F Identify properties of inverses of functions.

I Recognize functions and their properties from their graphs.

K Graph inverses of functions.

Vocabulary

inverse of a function
identity function

Function
- for every x-value there is exactly one y-value
- can't have repeating x-values
- vertical line test can only touch graph in one spot

yes function

Mental Math

What operation undoes each action?

a. adding $\frac{2}{3}$ to a number

b. multiplying a number by $\frac{\pi}{2}$

c. squaring a positive number

a) subtracting $2/3$
b) dividing by $\pi/2$
c) square root

Inverse of a function: the relation in which the components of all ordered pairs of the function are switched

* every function has an inverse, but not all inverses are functions.

Notation: the inverse of f(x) is denoted $f^{-1}(x)$

$$*\, f^{-1}(x) \neq \frac{1}{f(x)} \text{ even though } x^{-1} = \frac{1}{x}$$

Example 1: Let h = {(1, 1), (2, 4), (3, 9), (4, 16)}

a) Is h a function? Explain.

Yes, no repeating x-values

b) Describe the inverse of h.

h^{-1} {(1,1) (4,2) (9,3) (16,4)}

c) Is the inverse a function? Explain.

Yes, no repeating x-values

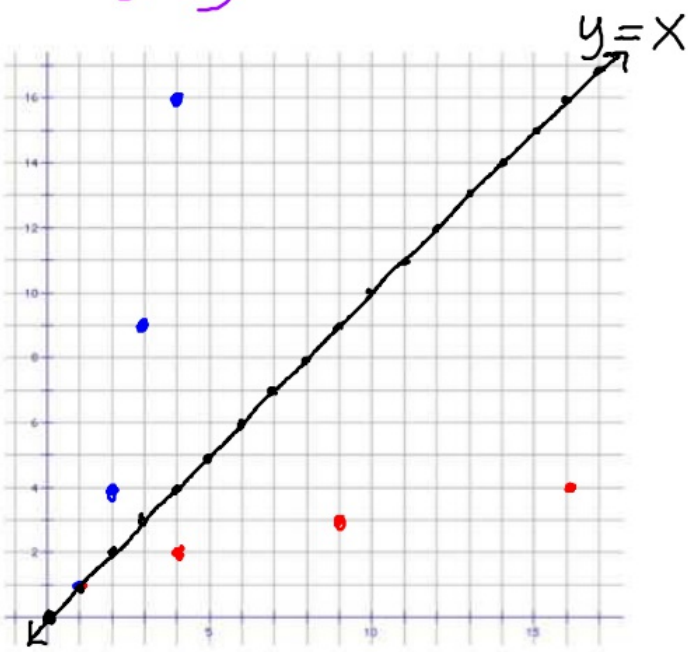
d) Describe h and its inverse in words.

h: Square x to get y

h^{-1}: Square root x to get y

e) Plot the points for h and its inverse.
What do you notice?

reflected over the line y=X



Example 2: Consider the function $y = 2(x+5)^2 - 1$.

a) Describe the graph of the function.

parabola, left 5, down 1, vertical stretch of 2,

opens up, vertex at $(-5,-1)$

b) Give an equation for the inverse of the function.

① Switch x & y

$x = 2(y+5)^2 - 1$

$+1 \qquad\qquad +1$

$\dfrac{x+1}{2} = \dfrac{2(y+5)^2}{2}$

$\dfrac{x+1}{2} = (y+5)^2$

② Solve for y

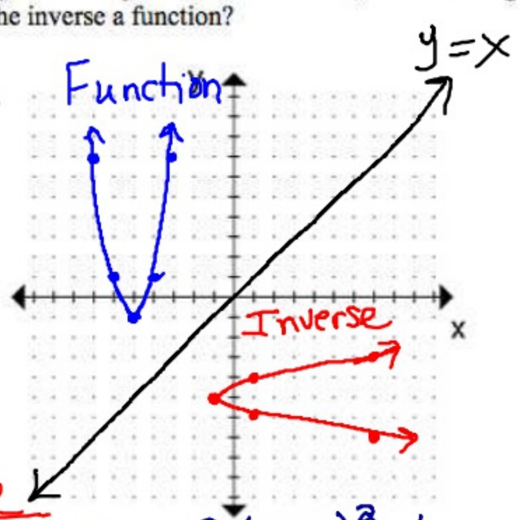
$\sqrt{\dfrac{x+1}{2}} = y+5$

$\boxed{\pm\sqrt{\dfrac{x+1}{2}} - 5 = y}$

Square root
left +1
down 5
horizontal stretch of 2
reflect over x-axis

c) Based on your answer to Part a, describe the graph of the inverse of the function. Is the inverse a function?

Function
$(-7,7)$
$(-6,1)$
$(-5,-1)$
$(-4,1)$
$(-3,7)$



Not a function,
fails vertical line test

$y_2 = \sqrt{((x+1)/2)} - 5$

$y_3 = -\sqrt{((x+1)/2)} - 5$

$y_4 = x$

Inverse
$(7,-7)$
$(1,-6)$
$(-1,-5)$
$(1,-4)$
$(7,-3)$

$y_1 = 2(x+5)^2 - 1$
2nd window
Tblstart = 0
ΔTbl = 1
2nd Graph

hyperbola
left 8, down 1
Stretch of 5

Example 3: Give an equation for the inverse of the function with equation $y = \frac{5}{(x+8)} - 1$

① Switch x & y ② Solve for y

$x = \frac{5}{y+8} - 1$ $\quad (5)\frac{1}{x+1} = \frac{y+8}{\cancel{5}}\cancel{(5)}$

$\frac{x+1}{1} = \frac{5}{y+8}$ $\quad \frac{5}{x+1} = y+8$

$$\boxed{\frac{5}{(x+1)} - 8 = y}$$

left 1, down 8
Stretch of 5

b) Is the inverse a function?

Yes, passes vertical line test

Inverse Functions and Composite Functions

Inverses of Functions Theorem

Given any two functions f and g, f and g are inverse functions if and only if $f(g(x)) = x$ for all x in the domain of g, and $g(f(x)) = x$ for all x in the domain of f.

Example 4: Use the inverse of Functions Theorem to determine whether f and g are inverses.

Does $f(g(x)) = g(f(x)) = x$?

$$f(x) = \frac{3x+1}{5-x} \qquad g(x) = \frac{5x-1}{x+3}$$

$f(g(x))$

$$\frac{\frac{3}{1}\left(\frac{5x-1}{x+3}\right) + \frac{1}{1}\frac{(x+3)}{(x+3)}}{\frac{5}{1}\frac{(x+3)}{(x+3)} - 1\left(\frac{5x-1}{x+3}\right)}$$

$$\frac{\frac{15x-3+x+3}{x+3}}{\frac{5x+15-5x+1}{x+3}}$$

$$\frac{\frac{16x}{x+3}}{\frac{16}{x+3}} \div$$

$$\frac{\cancel{16}x}{\cancel{x+3}} \cdot \frac{\cancel{x+3}}{\cancel{16}}$$

$= x$

$g(f(x))$

$$g\left(\frac{3x+1}{5-x}\right)$$

$$\frac{5\left(\frac{3x+1}{5-x}\right) - 1}{\frac{3x+1}{5-x} + 3}$$

$$\frac{\frac{15x+5}{5-x} - 1\frac{(5-x)}{(5-x)}}{\frac{3x+1}{5-x} + 3\frac{(5-x)}{(5-x)}}$$

$$\frac{\frac{15x+5-5+1x}{5-x}}{\frac{3x+1+15-3x}{5-x}}$$

$$\frac{\frac{16x}{5-x}}{\frac{16}{5-x}} \div$$

$$\frac{\cancel{16}x}{\cancel{5-x}} \cdot \frac{\cancel{5-x}}{\cancel{16}}$$

$= \boxed{x}$

f and g are inverses since $f(g(x)) = g(f(x)) = x$